# Reply to "Comment on 'Scaling of the linear response in simple aging systems without disorder' "

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The value of the nonequilibrium exponent $a$ is measured in the two-dimensional Ising model quenched to below criticality from the dynamical scaling of the zero-field-cooled and the intermediate susceptibility. Our results fully reconfirm the expected value $a=1/2$ but are inconsistent with the value $a=1/4$, advocated by Corberi, Lippiello, and Zannetti, Phys. Rev. E **72**, 028103 (2005).

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For simple magnets quenched to below their critical point, one generally expects, in the scaling regime where $t,s \gg t_{\text{micro}}$ and also $t-s \gg t_{\text{micro}}$ ($t_{\text{micro}}$ is a microscopic reference time), the following scaling behavior of the linear autoresponse function:

$$R(t,s) = \left. \frac{\delta\langle\phi(t)\rangle}{\delta h(s)} \right|_{h=0} \sim s^{-1-a} f_R(t/s), \tag{1}$$

where $\phi(t)$ is the order parameter at the observation time $t$ and $h(s)$ is the magnetic field at the waiting time $s$.

For some time, Corberi, Lippiello, and Zannetti (CLZ) [1,2] have advocated a phenomenological formula

$$a = \frac{n}{z}\left(\frac{d-d_L}{d_U-d_L}\right), \tag{2}$$

where $d_{U,L}$ are the upper and lower critical dimensions, respectively, and $n=1$ (2) for a scalar (vector) order parameter. On the other hand, the commonly accepted physical picture [3] of the aging (coarsening) process going on after the quench has it that ordered domains form very rapidly and that the aging comes from the movement of the domain walls between the ordered regimes. If this idea is combined with scaling arguments and if one takes into account that in certain systems (scalar systems such as the Ising model and referred to as class S) the spatial correlations decay exponentially while in others (such as the spherical model and referred to as class L) the spatial correlations decay algebraically, viz. $C_{\text{eq}}(\vec{r}) \sim |\vec{r}|^{-(d-2+\eta)}$, one obtains [4,5]

$$a = \begin{cases} 1/z & \text{for class S} \\ (d-2+\eta)/z & \text{for class L}, \end{cases} \tag{3}$$

where $z$ is the dynamical exponent. The result (3) has been reproduced in many studies. However, if Eq. (2) of CLZ should turn out to be correct that would also invalidate the simple physical picture of the coarsening process mentioned above. A good practical way to decide between Eq. (2) and the prediction (3) appears to be a study of the two-dimensional (2D) Ising model with a nonconserved order parameter and quenched to $T<T_c$ from a fully disordered initial state. Here $z=2$ is known [6] and from Eqs. (2) and (3) one has $a=1/4$ and $a=1/2$, respectively.

CLZ first arrived at Eq. (2) by analyzing numerical data [2] of the field-cooled susceptibility $\chi_{\text{ZFC}}(t,s)=\int_s^t du R(t,u)$. From a straightforward integration of Eq. (1), they obtained $\chi_{\text{ZFC}}(t,s) \sim s^{-a}$ and proceeded to extract $a$. However, it was pointed out that $\chi_{\text{ZFC}}(t,s)$ does not obey a simple scaling but rather there may be a further and dominant contribution coming from the upper limit of integration. For systems of class S, there are well-defined domains with domain walls whose thickness is small with respect to the domain size and one has [5]

$$\chi_{\text{ZFC}}(t,s) = \chi_0 + \chi_1 t^{-A} - s^{-a} f_M(t/s); \quad \text{class S}, \tag{4}$$

where $\chi_{0,1}$ are constants and $f_M(x) \geq 0$ is a scaling function. Indeed, it can be shown that $A=z^{-1}-\kappa$ [5], where the exponent $\kappa \geq 0$ describes the time-dependent scaling of the width of the domain walls $w(t) \sim t^{\kappa}$ [7]. In the 2D Ising model $\kappa=1/4$ is known [7], hence $A=1/4$ which explains the early result of CLZ but it becomes clear that the term naively expected from the scaling behavior (1) merely yields a finite-time correction. CLZ did not take into account the condition $t\text{-}s \gg t_{\text{micro}}$ for the validity of the scaling form (1) in their analysis [2] and hence have missed the leading time-dependent term in Eq. (4).

On the other hand, for systems of class L, we argued previously that because of the long-range correlations, the effective width of the domains should be the same as their linear size which leads to $A=0$ [5]. CLZ [1] point out correctly that for both classes S and L the constant $\lim_{t\to\infty} \chi_{\text{ZFC}}(t,s)$ (with $x=t/s$ fixed) and which equals $\chi_0$ for class S and $\chi_0+\chi_1$ for class L can be related to the equilibrium magnetization through the static fluctuation-dissipation theorem. In summary one has, with $a$ given by Eq. (3)

$$\chi_{\text{ZFC}}(t,s) = \begin{cases} \dfrac{1}{T}(1-m_{\text{eq}}^2) + \chi_1 t^{-A} - s^{-a} f_M(t/s) & \text{for class S} \\ \dfrac{1}{T}(1-m_{\text{eq}}^2) - s^{-a} f_M(t/s) & \text{for class L}. \end{cases} \tag{5}$$

 

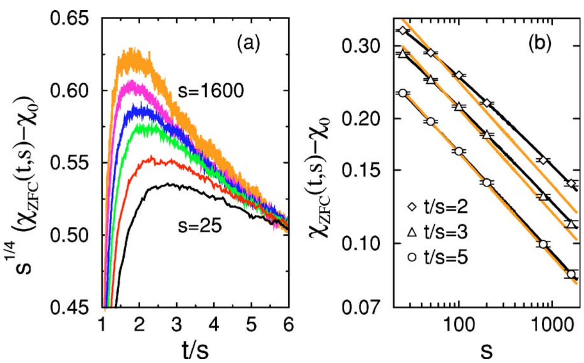


FIG. 1. (Color online) (a) Scaling plot of the reduced field-cooled susceptibility $\chi_{ZFC}(t,s)-\chi_0$ over against $t/s$ in the 2D Ising model quenched to $T=1.5$, for waiting times $s=25,50,100,200,800,1600$ (from bottom to top). (b) Comparison of the data with a fit $\chi_{ZFC}(t,s)-\chi_0=a_0 s^{-1/4}+a_1 s^{-1/2}$ (black lines) and a fit $\chi_{ZFC}(t,s)-\chi_0=a_0 s^{-1/4}$ (gray lines), for several values of $t/s$. For clarity the data for $t/s=2$ and for $t/s=3$ have been shifted upwards by a factor 1.4 and 1.2, respectively.

In their comment, CLZ [1] now argue that the exponents $A$ and $a$ were really not distinct and that one rather had $a=A$. From now on, we concentrate on systems of class S and furthermore on the 2D Ising model quenched below $T_c$. From numerical simulations, CLZ reproduce once more that $\chi_{ZFC}(t,s)-\chi_0 \sim s^{-1/4}$, as generally expected. But they do not consider explicitly the corrections to that leading scaling behavior and merely claim, but do not prove, the absence of the scaling corrections of order $s^{-a}$, which are expected from Eq. (5).

Indeed, the scaling behavior of $\chi_{ZFC}$ is not as simple as claimed by CLZ. We show this in Fig. 1(a), with data obtained from the standard heat-bath method. It can be seen that the collapse in this scaling plot is far from complete which strongly indicates the presence of sizeable finite-time corrections. It must be noted that in CLZ [1] the corresponding data are plotted in such a way that the finite-time corrections are not discernable. However, focusing instead on the interesting part around the maximum, as we do in Fig. 1(a), permits one to reveal their existence. The origin of these corrections becomes clear in Fig. 1(b) where we compare our data to the asymptotic form $\chi_{ZFC}(t,s)-\chi_0=a_0 s^{-1/4}+a_1 s^{-1/2}$, for several values of $t/s$. While taking into account both terms does reproduce the data well, we included for comparison also the fits where $a_1=0$ was fixed by hand, as suggested by CLZ. It is clear that the data, which display a pronounced curvature, can only be fitted if $a_1<0$, as expected from Eq. (5). Indeed, the best fits yield $a_0=0.68$ and $a_1=-0.35$ for $t/s=2$, $a_0=0.62$ and $a_1=-0.19$ for $t/s=3$, and $a_0=0.54$ and $a_1=-0.06$ for $t/s=5$ in excellent quantitative agreement with Eq. (5). We conclude that there is no indication in favor of an absence of the last term in Eq. (5), hence $a>A$ which provides clear counterevidence against the proposal of CLZ.

We had already given earlier an analysis of the scaling of the thermoremanent magnetization (where terms of order $s^{-A}$ do not occur). By taking into account also the leading finite-time correction we found full compatibility with $a=1/2$ but inconsistency with $a=1/4$ [4].

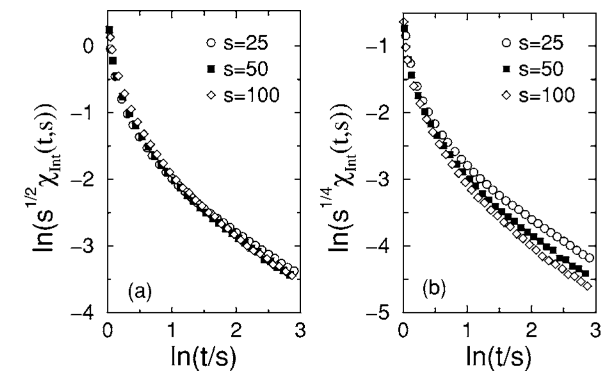


FIG. 2. Scaling plot of the intermediate susceptibility $\chi_{Int}(t,s)$ for (a) the assumed value $a=1/2$ and (b) the assumed value $a=1/4$. Error bars are smaller than the symbol sizes.

We may also show that indeed $a=1/2$ in the 2D Ising model through the so-called intermediate integrated response [5,8]

$$\chi_{Int}(t,s) = \int_{s/2}^{s} du R(t,u) \sim s^{-a} f_{Int}(t/s), \qquad (6)$$

which has the advantage that the leading term $\sim t^{-A}$ which dominates over the scaling term in $\chi_{ZFC}$ is absent. In Fig. 2 we examine the scaling of $\chi_{Int}(t,s)$ and try to achieve a collapse for $a=1/2$ and $1/4$. While there is a nice collapse for $a=1/2$ already for relatively small values of the waiting time (unless $t/s$ is too close to unity), the data fail to collapse for $a=1/4$. Again, this is fully consistent with $a=1/2$ but excludes $a=1/4$.

Turning to systems of class L, CLZ nicely reconfirmed the expected scaling forms Eqs. (3) and (5). However, there is nothing in their test with contradicts the simple scaling we used in [5] to obtain $A=0$.

In conclusion, having reexamined the scaling of some integrated susceptibilites, we have shown that the scaling of $\chi_{ZFC}(t,s)$ does indeed contain at least two important contributions, see Eq. (5), which is against the proposal of CLZ. We stress that a simple demonstration of scaling of $\chi_{ZFC}$ is not enough to be able to reliably know which of the exponents $A$ or $a$ is measured. It is more safe to study a quantity such as the intermediate susceptibility, which does not suffer from this difficulty. Applied to the Ising model quenched to $T<T_c$, our results fully confirm Eq. (3) but disagree with Eq. (2).

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[1] F. Corberi, E. Lippiello, and M. Zannetti, preceding comment, Phys. Rev. E **72**, 028103 (2005), and references therein.
[2] F. Corberi, E. Lippiello, and M. Zannetti, Eur. Phys. J. B **24**, 359 (2001); Phys. Rev. E **68**, 046131 (2003).
[3] See the reviews by A. J. Bray and J. P. Bouchaud in *Soft and Fragile Matter*, edited by M. E. Cates and M. R. Evans (IOP, Bristol, 2000).
[4] M. Henkel, M. Paessens, and M. Pleimling, Europhys. Lett. **62**, 664 (2003).
[5] M. Henkel, M. Paessens, and M. Pleimling, Phys. Rev. E **69**, 056109 (2004).
[6] A. J. Bray, Adv. Phys. **43**, 357 (1994).
[7] D. B. Abraham and P. J. Upton, Phys. Rev. B **39**, 736 (1989).
[8] M. Pleimling and A. Gambassi, Phys. Rev. B **71**, 180401(R) (2005).